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RESERVING BY DETAILED CONDITIONING ON INDIVIDUAL CLAIM

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Abstract. The estimation of claim reserves presents an important task for insurance companies to predict their liabilities. Recently, reserving method of individual claim have attracted aplenty of interest in the actuarial literature, which overcome some deficiency of aggregated claim method. This paper explores the Reserving by Detailed Conditioning (RDC) method using all of claim information for reserving with individual claim. By using the original data from general insurance, we demonstrate this method and compare it to some existing method.

Keywords: claim reserving, individual claim, claim information, RDC.

INTRODUCTION

In some types of insurance, sometimes payments of claim are not finalized at the moment evaluation. Based on that reason, insurance companies must have enough funds as reserves. Generally, reserving methods are based on aggregated data presented on run-off triangle[1][2]. The large data sets with all of the claim information of the policyholder are summarized into small sample design. Therefore, a lot of detailed claim information of the policyholder is eliminated[3][4].

Aggregated data on reserving method will result in higher variance and probably even biased estimate, due to the elimination of claim information[5]. Recently, a lot of research is developed to find reserving methods that are able to process the full data set. One of the methods is Reserving by Detailed Conditioning (RDC) proposed by Rosenlund[6]. RDC method is a reserve estimation method claim designed for individual claims in discrete time. Reserving process using this method is done without aggregating data. Therefore, the detailed information of each claim can be included as a condition of calculation. For knowing the performance of RDC method, we will use data of liability insurance policies from an Indonesian general insurance company as a case study.

CONSTRUCTION OF THE DATA

Individual Claim Information

In the RDC method, the claim information of N claims with each claim k ($k = 1, 2, \cdots, N$) on continuous time are discretized. In the following we denote

$$\{i(k), W(k), F(k), Y(k,1), \cdots, Y(k, F(k))\}, \tag{1}$$

as a set of individual claim information. The notation of $i(k)$ denotes occurrence period of claim k with $i(k) \in \{1, 2, \cdots, n\}$, $W(k)$ denotes reporting period of claim k, $F(k)$ denotes closing period of claim k, and $Y(k)$ denotes payments of claim k on development period j, with $j \in \{1, 2, \cdots, F(k)\}$. If a claim is reported at the occurrence period then set the value of $W(k) = 1$. Likewise if a claim is closed at the occurrence period then set the value of $F(k) = 1$.

The individual claim information as described by (1) cannot be directly used as condition of claim reserve estimation. Reserving process becomes inefficient if it is used. Therefore, we have to create a claim information representative called claim characteristics to simplify the calculation process.

Claim Characteristics

The characteristics of claim in RDC method is divided into three parts, i.e. claim length $L(k)$, reporting delay $W(k)$, and quantile interval number of cumulative payments $Q_t(k)$.

Claim Length

The claim length is the time duration since claim reported up to claim can be finalized. The claim length is defined by

$$L(k) = F(k) - W(k) + 1. \tag{2}$$

Reporting Delay Period

Claims that have the same reporting delay period may have a similar development pattern. Based on that reason, reporting delay period is used as the one of claim characteristics. In this case, we set w_0 as a maximum limit of $W(k)$. A claim is considered late to be reported at insurance company if $W(k) \geq w_0$. Therefore, we define $\min(W(k), w_0)$ as a boundary of reporting delay of individual claim.

Quantile Interval Number of Cumulative Payment

The claims will be divided into several of quantile intervals based on information from the cumulative payments. The cumulative payment, $H(k,t)$, for claim k on the time t starting from the reporting period is defined by

$$H(k,t) = \sum_{h=1}^{t} Y(k, h + W(k) - 1), \quad t \in \{0, 1, \cdots, n - i(k) - W(k) + 2\}. \tag{3}$$

For computing the quantile intervals, we have to sort in ascending order $H(k,t)$ with $L(k) > t$ in a set of $H(\cdot, t)$. We split $H(\cdot, t)$ (for given t) into q_0 quantile intervals. The number of quantile intervals q_0 is fixed in advance. If we denote the boundaries of quantile as $\{h_{t,1}, h_{t,2}, \cdots, h_{t,q_0}\}$, we can create the quantile intervals of $H(\cdot, t)$ as $[0, h_{t,1}], (h_{t,1}, h_{t,2}], \cdots, (h_{t,q_0-1}, h_{t,q_0}]$. The interval number to which $H(k,t)$ belongs is indicated by $Qt(k)$,

$$Q_t(k) = \text{quantile interval number of } H(k,t) \text{ with } L(k) > t, \quad Q_t(k) \in \{1, 2, \cdots, q_0\} \tag{4}$$

for $t > 0$. When $t = 0$ the claim has not been reported yet, thus there is no information on the cumulative payments. Consequently, $Q_0(k) = 1$.

After we have $Q_t(k)$ for each t and k, we need do modification in the grouping of quantile intervals so that each group per q, t, w contains at least one finalized claim for $q \in \{1, 2, \cdots, q_0\}$, $t \in \{1, 2, \cdots, n\}$, and $w \in \{1, 2, \cdots, w_0\}$. If a group initially contains only open claims it is merged with the group with the same t, w and nearest lower q with finalized claims, if such a group exists. If not, it is merged with group with the same t, w and the nearest higher q with finalized claims. If not even such a group exists, $Q_t(k)$ is set to 1.

RESERVING BY DETAILED CONDITIONING

There are two steps to calculate claim reserve estimation using RDC method. First, we compute an estimate of the probability distribution of claim length. Second, we compute an estimate of the expected value of the payment amount per future development period. The reserve can be obtained by multiplication between the first and the second one.

Estimation of Claim Length Probability

We define the estimation of claim length probability as

$$\hat{p}_\lambda(t,q,w) = \hat{r}_\lambda(t,q,w) \prod_{m=t+1}^{\lambda-1} (1 - \hat{r}_m(t,q,w)). \tag{5}$$

for $q \in \{1,2,\cdots,q_0\}$ and $w \in \{1,2,\cdots,w_0\}$ with the given $0 \le t \le n-1$ and $t+1 \le \lambda \le n$. The notation of $\hat{r}_\lambda(t,q,w)$ is the estimated probability of finalized of claim in the following period. This is done by the following process

$$\begin{cases} \hat{r}_n(t,q,w) = 1, \\ \hat{r}_\lambda(t,q,w) = \frac{I_\lambda{}^F(t,q,w)}{J_\lambda(t,q,w)}, \quad \text{for } \lambda < n, \end{cases} \tag{6}$$

where

$I_\lambda^F(t,q,w)$ is the number of claims closed given $L = \lambda, Q_t = q, \min(W,w_0) = w$,
$J_\lambda(t,q,w)$ is the number of claims reported given $L \ge \lambda, Q_t = q, \min(W,w_0) = w$.

Estimation of Mean Payment

The estimation of mean payment is obtained via combination of payments from opened claims and closed claims. Therefore, the observation of claims is divided into three stage, i.e. the observation of closed claims, the observation of opened claims, and the observation of claims that are opened in the previous period then closed in the following period.

For closed claims, the observation of claims is done in the period $h \le n-i-W+2$ with the given $0 \le t \le n-1$, $t+1 \le \lambda \le n$, and $t+1 \le h \le \lambda$. For all $q \in \{1,2,\cdots,q_0\}$ and $w \in \{1,2,\cdots,w_0\}$, we calculate $I_{\lambda h}^F(t,q,w)$ and $Y_{\lambda h}^F(t,q,w)$, where

$I_{\lambda h}^F(t,q,w)$ is the number of closed claim given $L \le n-i-W+2, L = \lambda, Q_t = q, \min(W,w_0) = w$,
$Y_{\lambda h}^F(t,q,w)$ is the number of claim payments given $L \le n-i-W+2, L = \lambda, Q_t = q, \min(W,w_0) = w$.

For opened claims, the observation of claims is done in the period $t \le n-2$ with the given $0 \le t \le n-2$, $t+1 \le r \le n-1$, and $t+1 \le h \le r$. For all $q \in \{1,2,\cdots,q_0\}$ and $w \in \{1,2,\cdots,w_0\}$, we calculate $I_r^\circ(t,q,w)$ and $Y_{rh}^\circ(t,q,w)$, where

$I_r^\circ(t,q,w)$ is the number of closed claim given $n-i-W+2 = r, L > r, Q_t = q, \min(W,w_0) = w$,
$Y_{rh}^\circ(t,q,w)$ is the number of claim payments given $n-i-W+2 = r, L > r, Q_t = q, \min(W,w_0) = w$.

For the claims that are opened in the previous period (on time r) but can closed in the following period (at $L = \lambda$), we calculate

$$I_{r\lambda}^\circ(t,q,w) = \frac{\hat{p}_\lambda(t,q,w)}{\hat{p}_{r+1}(t,q,w) + \cdots + \hat{p}_n(t,q,w)} I_r^\circ(t,q,w), \tag{7}$$

$$Y_{r\lambda h}^\circ(t,q,w) = \beta_{rh}(t,q,w) \frac{Y_{\lambda h}^{r+1}(t,q,w)}{I_\lambda^{r+1}(t,q,w)} I_{\lambda r}^\circ(t,q,w), \tag{8}$$

with

$$\beta_{rh}(t,q,w) = Y^{\circ}_{rh}(t,q,w)\left(\sum_{v=r+1}^{n} Y^{r+1}_{vh}(t,q,w)\frac{I^{\circ}_{rv}(t,q,w)}{I^{(r+1)}_{v}(t,q,w)}\right)^{-1}. \tag{9}$$

If $\left(\sum_{v=r+1}^{n} Y^{r+1}_{vh}(t,q,w)\frac{I^{\circ}_{rv}(t,q,w)}{I^{(r+1)}_{v}(t,q,w)}\right)^{-1} = 0$, then the value of $\beta_{rh}(t,q,w)$ will be undefined. Therefore, the alternative way to calculate the value of $Y^{\circ}_{r\lambda h}(t,q,w)$ is

$$Y^{\circ}_{r\lambda h}(t,q,w) = \frac{\hat{p}_{\lambda}(t,q,w)}{\hat{p}_{r+1}(t,q,w)+\cdots+\hat{p}_{n}(t,q,w)} Y^{\circ}_{rh}(t,q,w). \tag{10}$$

The estimation of mean payment is obtained using calculation of backward recursive. The backward recursive calculation on period $r=\lambda$ up to period $r=h$ use the initial value,

$$\begin{cases} I^{(\lambda)}_{\lambda}(t,q,w) = I^{F}_{\lambda}(t,q,w), \\ Y^{(\lambda)}_{\lambda h}(t,q,w) = Y^{F}_{\lambda h}(t,q,w). \end{cases} \tag{11}$$

The equation of recursive calculation for $r=\lambda-1,\lambda-2,\cdots,h$ can be expressed by

$$\begin{cases} I^{(r)}_{\lambda}(t,q,w) = I^{(r+1)}_{\lambda}(t,q,w) + I^{\circ}_{r\lambda}(t,q,w) \\ Y^{(r)}_{\lambda h}(t,q,w) = Y^{(r+1)}_{\lambda h}(t,q,w) + Y^{\circ}_{r\lambda h}(t,q,w), \end{cases} \tag{12}$$

After the recursive calculation up to period of h is completed, we can obtain the final estimation of mean payment that is expressed by

$$\hat{\mu}_{\lambda h}(t,q,w) = \frac{Y^{(h)}_{\lambda h}(t,q,w)}{I^{(h)}_{\lambda}(t,q,w)}, \tag{13}$$

with $h=t+1,\cdots,n$ and $\lambda=h,\cdots,n$.

Estimation of Claim Reserves

The estimation of claim reserves is divided into two types, i.e. IBNR and RBNS. The total estimation of claim reserves in the occurrence period $i\in\{1,2,\cdots,n\}$ can be expressed by

$$\hat{R}_i = \hat{R}^I_i + \hat{R}^R_i \tag{14}$$

with $\hat{R}^I_i$ denotes IBNR claims reserves and $\hat{R}^R_i$ denotes RBNS claim reserves.

IBNR Reserve

IBNR reserve is a reserve of claim that have not yet reported to the insurance company. Given $W=w$, for each $w\in\{1,2,\cdots,w_0\}$ the estimated IBNR reserves is

$$\hat{R}(0,1,\min(w,w_0)). \tag{15}$$

The estimation of the number of claims per occurrence period $i\in\{2,3,\cdots,n\}$ is done using the well-known chain ladder method by utilizing the data claims that have been reported to the company. Thus the estimated IBNR claim reserves in the occurrence period $i\in\{2,3,\cdots,n\}$ can be expressed by

$$\hat{R}^I_i = \sum_{w=n-i+2}^{n} \hat{A}_{iw}\hat{R}(0,1,\min(w,w_0)), \tag{16}$$

where

$$\begin{cases} A_{iw} \text{ is the number of claims reported in development period } w, \\ \hat{A}_{iw} \text{ is the estimation of } A_{iw} \text{ using chain ladder method for } w\ge n-i+2. \end{cases}$$

RBNS Reserve

RBNS reserve is a reserve of claim that have been reported to the insurance company but that have not yet finalized. We denote RBNS reserve at claim period i as following

$$\hat{R}(n-i-W+2, Q_{n-i-W+2}, W \wedge w_0). \tag{17}$$

Suppose for the claim period i

$I^o(i,q,w)$is the number of opened claims given $W=w$ and $Q_{n-i-W+2}=q$.

then RBNS reserves for the claim period i is

$$\hat{R}_i^R = \sum_{w=1}^{n-i+1} \sum_{q=1}^{q_0} I^o(i,q,w)\hat{R}(n-i-w+2, q, \min(w, w_0)), \tag{18}$$

CASE STUDY

Data

We have a data of liability insurance claim from general insurance company in Indonesia. The data starts from January 2014 to December 2014. The data claim consist of the information of ID claim, the occurrence period, the reporting period, the period of claim closed, and the payment data. The information of claim represented in the form of a run-off triangle individual[7]. The concept of run-off triangle individual similar with run-off triangle aggregate but in the run-off triangle aggregate the data set of claim is not summarized.

The number of liability insurance claim which taken as samples is 270 claims. For the data, we describe reporting delay period, claim length, and descriptive statistics in Table 1, 2, and 3.

TABLE 1. Reporting Delay of Liability Insurance Claim

W	1	2	3	4	5	6	7	8	9	10	11	12
Number of Claims	13	100	84	33	22	12	4	0	2	0	0	0
	4,81%	37,04%	31,11%	12,22%	8,15%	4,44%	1,48%	0,00%	0,74%	0,00%	0,00%	0.00%

TABLE 2. Claim Length of Liability Insurance Claim

L	1	2	3	4	5	6	7	8	9	10	11	12
Number of Claims	183	47	9	6	4	10	7	1	3	0	0	0
	67,78%	17,41%	3,33%	2,22%	1,48%	3,70%	2,59%	0,37%	1,11%	0,00%	0,00%	0,00%

TABLE 3. Descriptive Statistics

	Claim Payments (IDR)
Mean	4.313.271
Standard Deviation	7.540.700
Minimum	3.000
Maximum	87.877.500
Sum	1.483.765.162
Number Observation	270

Result

In this paper, the calculation of liability insurance claims reserve estimation is divided into two types, i.e. the estimation of IBNR reserve and the estimation of RBNS reserve. Before starting calculates reserve estimation of liability insurance claim, we must determine the value of w_0 and q_0 based on descriptive statistics. Based on table 1, we assume that a claim is reported late if $W \geq 3$. Therefore, we set $w_0 = 3$. For q_0 we set $q_0 = 3$. It because we separate the data of liability insurance claim into three types based on cumulative payments, i.e. small, medium, and large claims. The following shows the result of IBNR, RBNS, and total reserve estimation of liability insurance claims with $w_0 = 3$ and $q_0 = 3$ for occurrence period $i \in \{2, 3, \cdot, n\}$ in Indonesian Rupiahs (IDR).

TABLE 4. Results

Occurrence Period	IBNR (IDR)	RBNS(IDR)
2	0	0
3	0	5.659.850
4	0	28.299.250
5	3.978.013	5.802.417
6	3.264.011	10.495.867
7	4.940.165	278.750
8	12.489.228	1.103.438
9	27.846.314	27.343.175
10	57.873.259	17.929.522
11	50.898.626	8.152.534
12	140.414.913	0
Total	301.704.529	105.064.803

For the same data, the following will be displayed the claim reserve estimation results using RDC method with some combination of the characteristics of the claim and their MSEP value. We use BICH method to calculate the MSEP[6].

TABLE 5. Estimation MSEP

Method	Total Reserve	Estimation MSEP
RDC with $w_0 = 1, q_0 = 1$	396.752.066	104.427.335
RDC with $w_0 = 1, q_0 = 3$	379.924.051	101.705.350
RDC with $w_0 = 5, q_0 = 3$	442.682.602	101.750.118
RDC with $w_0 = 8, q_0 = 3$	439.645.399	106.139.134
RDC with $w_0 = 10, q_0 = 3$	439.645.399	106.774.598
RDC with $w_0 = 3, q_0 = 3$	406.769.331	100.698.631
RDC with $w_0 = 3, q_0 = 1$	446.567.676	103.539.107
RDC with $w_0 = 3, q_0 = 8$	435.555.734	102.561.981
Chain Ladder	392.628.754	237.605.914
Bornhuetter-Ferguson	449.392.103	118.332.711

Based on Table 5, we know that the claim reserve estimation using RDC with a combination of $w_0 = 3$ and $q_0 = 3$ gives a fewer value of MSEP estimation than the other combination. As in [6], the right combination of claim characteristics is indicated by a small value of MSEP. Thus, we can say that the combination of claim characteristics $w_0 = 3$ and $q_0 = 3$ is more appropriately used than the other combination of claim characteristics to estimate claim reserves of liability insurance data. In addition, based on Table 5, we know that RDC method gives a fewer value of MSEP estimation than Chain ladder and Bornhuetter-Ferguson. From these results, it shows that RDC method gives more accurate results than chain ladder and Bornhuetter-Ferguson for the estimation of claims reserve on liability insurance data.

CONCLUSIONS

The utilization of claim information as a detailed condition is a new breakthrough on estimation of claim reserves. Therefore, RDC method able to estimate of claim reserves with detailed individual claim infromation. We transferred claim information into claim characteristics so that easier to use on calculation process. We use claim length, reporting delay, and cumulative payments as characteristics of the claim. We perform calculations RDC method with data from general insurance company. We compared the results of this method by Chain ladder and Bornhutter-Ferguson. The RDC method looks to have the smallest MSEP which means that the best method in the case studies.

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